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won four points, the chance that A wins two successive points is $40a^6b^4$. The chance that B wins two successive points is $40a^4b^6$, the chance that each win a point is $80a^5b^5$, and so on.

Let p=A's chance of winning the first game, and q=B's chance of winning the first game.

$$\therefore p=a^4+4a^4b+10a^4b^2+20a^5b^3+40a^6b^4+80a^7b^5+160a^8b^6+\ldots\ldots$$
$$=a^4+4a^4b+10a^4b^2+20a^5b^3(1+2ab+4a^2b^2+8a^3b^3+\ldots\ldots)$$
$$=a^4+4a^4b+10a^4b^2+\frac{20a^5b^3}{1-2ab}=\frac{a^4}{1-2ab}(1+4b+10b^2-2ab-8ab^2).$$

Similarly, $q=\dfrac{b^4}{1-2ab}(1+4a+10a^2-2ab-8a^2b)$.

The chance that A wins a set in ten games is

$$p^6+6p^6q+\frac{6.7}{2!}p^6q^2+\frac{6.7.8}{3!}p^6q^3+\frac{6.7.8.9}{4!}p^6q^4$$
$$=p^6+6p^6q+21p^2q^2+56p^6q^3+126p^6q^4.$$

The chance that B wins the set in ten games is $q^6+6q^6p+21q^6p^2+56q^6p^3+126q^6p^4$.

The chance that each wins five games is $\dfrac{10!}{5!5!}p^5q^5=252p^5q^5$.

Let P be A's and Q be B's chance of winning the first set. Then as before

$$P=p^6+6p^6q+21p^6q^2+56p^6q^3+126p^6q^4+252p^6q^5+504p^8q^6$$
$$+1008p^9q^7+2016p^{10}q^8+\ldots\ldots$$
$$=p^6+6p^6q+21p^6q^2+56p^6q^3+126p^6q^4+252p^7q^5(1+2pq$$
$$+4p^2q^2+8p^3q^3+\ldots\ldots)$$
$$=p^6+6p^6q+21p^6q^2+56p^6q^3+126p^6q^4+\frac{252\,p^7q^5}{1-2pq}$$
$$=\frac{p^6}{1-2pq}(1+6q+21q^2+56q^3+126q^4-2pq-12pq^2-42pq^3-112pq^4).$$
$$Q=\frac{q^6}{1-2pq}(1+6p+21p^2+56p^3+126p^4-2pq-12p^2q-42p^3q-112p^4q).$$

The chance that A wins three sets out of five and thus wins the match is $P_1=P^3+3P^3Q+6P^3Q=P^3(1+3Q+6Q^2)$

The chance that B wins the match is $Q_1=Q^3(1+3P+6P^2)$.

MISCELLANEOUS.

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142. Proposed by R. A. WELLS, Bellevue College, Bellevue, Neb.

Find a general expression for the value of θ such that when θ is one of the acute angles of a right triangle, the three sides of the triangle will be commensurable.

I. Solution by O. W. ANTHONY, Head of Mathematical Department, DeWitt Clinton High School, New York City.

Without assuming the triangle to be right, we start with the equation

$$c^2=a^2+b^2-2ab\cos C,\ \therefore a=b\cos C\pm\surd(b^2\cos^2C-b^2+c^2).$$

Let $b^2\cos^2C-b^2+c^2=1^2$. Then $c^2=1^2+b^2\sin^2C$. Assume $1=x^2-y^2$, and $b\sin C=2xy$. Then $a=\surd(b^2-4x^2y^2)\pm(x^2-y^2)$.

Let $b^2-4x^2y^2=s^2$, $b+2xy=ks$, and $b-2xy=s/k$. Then $b=s(k^2+1)/2k$, $xy=s(k^2-1)/4k$.

Taking $s=4mk$ we obtain the following formulae:

$$b=2m(k^2+1),\quad c=x^2+m^2(k^2-1)^2/x^2,$$
$$a=4mk\pm[x^2-m^2(k^2-1)^2/x^2],\quad C=\cos^{-1}[2k/(k^2+1)].$$

As an example, assume $x=3$, $m=3$, $k=2$. We find $a=24$, $b=30$, $c=18$, $\cos C=\frac{4}{5}$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

m^2+n^2, m^2-n^2, $2mn$ are the sides of any right triangle. Let $m=pn$.

$\therefore n^2(p^2+1)$, $n^2(p^2-1)$, $2n^2p$ are the sides.

The equation for the general value of θ is

$$n^2(p^2-1)\cos\theta+2n^2p\sin\theta=n^2(p^2+1).$$

Let $\sin\beta=\dfrac{n^2(p^2-1)}{n^2(p^2+1)}$, $\cos\beta=\dfrac{2n^2p}{n^2(p^2+1)}$, then the equation becomes

$$\cos\beta\cos\theta+\sin\beta\sin\theta=1,.\ \cos(\theta-\beta)=1.$$

$\therefore\theta-\beta=2\pi n_1$, $\theta=2\pi n_1+\beta$.

As θ must be less than $\frac{1}{2}\pi$, the general value of θ is $\theta=\beta$.

Also solved by J. B. Greggs, M. Sc., C. E., Senecaville, Ohio.

PROBLEMS FOR SOLUTION.

ALGEBRA.

192. **Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, O.**

What is the difference between the squares of the two *infinite* continued fractions $\left(3+\dfrac{1}{6+\text{etc.}}\right)$ and $\left(2+\dfrac{1}{4+\text{etc.}}\right)$?

193. **Proposed by SAUL EPSTEEN, Ph. D., Chicago, Ill.**

Professor Goursat states (*Transactions of the American Mathematical Society*, January, 1904, p. 111) that if $a_1, a_2, \ldots\ldots, a_n$; $h_1, h_2, \ldots\ldots, h_n$ are two sequences, the h's being all positive, then $\sum\dfrac{a_i^2}{h_i^2}\geqq\dfrac{(\Sigma a_i)^2}{\Sigma h_i}$. Prove this.